# 3D Image Processing
# Lab Assignment 1
# Basic Concepts

Due: 04 November 2014, 9:30am

## Chapter 2: [15 Points]

### 1- Textbook Problem 2.7 (3 points)

Suppose that the flat area with center at $(x_0, y_0)$ is illuminated by a light source with intensity distribution

$$i(x, y) = Ke^{-[(x-x_0)^2 + (y-y_0)^2]}.$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0, and let $K = 255$. If the resulting image is digitized with $k$ bits of intensity resolution, and the eye can detect an abrupt change of eight shades of intensity between adjacent pixels, what value of $k$ will cause visible false contouring?

### 2- Textbook Problem 2.8 (4 points)

Sketch the image in Problem 2.7 for $k = 2$. Explain how you did it. you can do it in Matlab/Octave or Grapher or any other software of your choice as long as you explain how you model the problem in the software.

### 3- Textbook Problem 2.17 (4 points)

Give the condition(s) under which the $D_8$ distance between two points p and q is equal to the shortest 8-path between these points. Is the path unique?

### 4- Textbook Problem 2.19 (4 points)

The median, $\zeta$, of a set of numbers is such that half the values in the set are below $\zeta$ and the other half are above it. For example, the median of the set of values $2, 3, 8, 20, 21, 25, 31$ is 20. Show that an operator that computes the median of a subimage area, **S**, is nonlinear.

# Chapter 3: [25 Points]

## 5- Textbook Problem 3.8 (5 points)

In some applications it is useful to model the histogram of input images as Gaussian probability density functions of the form

$$p_r(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-m)^2}{2\sigma^2}},$$

where $m$ and $\sigma$ are the mean and standard deviation of the Gaussian PDF. The approach is to let $m$ and $\sigma$ measure of average intensity and contrast of a given image. What is the transformation function you would use for histogram equalization?

## 6- Textbook Problem 3.14 (5 points)

The following images are quite different, but their histograms are the same. Suppose that each image is blurred with a 3x3 averaging mask.
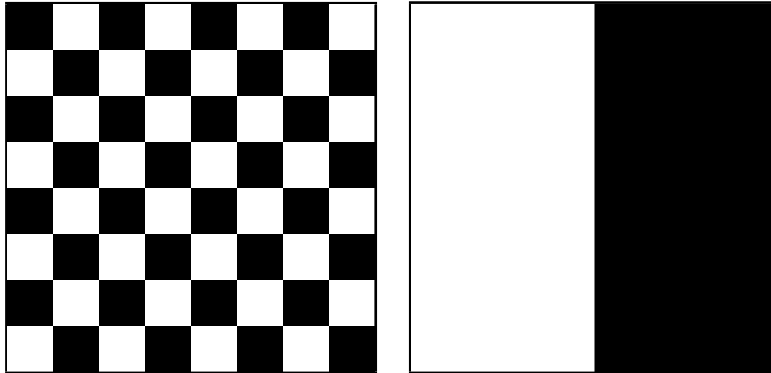


(a) Would the histograms of the blurred images still be equal? Explain why.

(b) If your answer is no, sketch the two histograms.

** Images can be generated in Matlab with the following instructions:

- a = (checkerboard(50, 4, 4) > 0.5);

- b = [ones(400,200), zeros(400,200)];

## 7- Textbook Problem 3.29 (6 points)

(a) Show that the magnitude of the gradient given in Eq. (3.6.11) (shown below) is an isotropic operation.

$$M(x, y) = |\nabla f| = \sqrt{g_x^2 + g_y^2}$$

(b) Show that the isotropic property is lost in general if the gradient is computed using Eq. (3.6.12):

$$M(x, y) \approx |g_x| + |g_y|$$

## 8- (9 points)

Suppose we have an $16 \times 16$ checkerboard image with $10 \times 10$ squares (total size $160 \times 160$). Apply the following filters in Matlab/Octave/C++/etc on the image and show the pictures:

(a) 6x6 box filter;

(b) 10x10 box filter;

(c) 14x14 box filter;

(d) 6x10 box filter.

Explain why the blurring looks stronger with $10 \times 10$ filter although it is smaller than the $14 \times 14$ filter. Justify your observation in spatial as well as frequency domain.

# Chapter 4: [30 Points]

## 9- (15 points)

Consider a continuous image I(x,y) as a $16 \times 16$ checkerboard with $1cm \times 1cm$ squares.

(a) What is the main period of I(x,y)?

(b) What is the dominant frequency of I(x,y)?

Draw a diagram similar to Fig. 4.6 and answer the following questions:

(c) What would the sampled image and its Fourier transform look like in general if I(x,y) is sampled at a rate higher than the Nyquist rate?

(d) What would the sampled image and its Fourier transform look like in general if I(x,y) is sampled at a rate lower than the Nyquist rate?

(e) What would the sampled image and its Fourier transform look like in general if I(x,y) is sampled at the Nyquist rate with samples taken at $t = 0, \Delta T, 2\Delta T, \cdots$

(f) How does the image look like in frequency domain? Compute and draw it on Matlab/Octave/C++/etc using a high density sampling.

(g) Draw sampled images and their frequency domain equivalents for sampling rates of 0.5N, N, and 2N, where N is the Nyquist rate.

## 10- Textbook Problem 4.26 (9 points)

(a) Show that the Laplacian of a continuous function $f(t, z)$ of continuous variables $t$ and $z$ satisfies the following Fourier transform pair [see Eq. (3.6.3) for a definition of the Laplacian]:

$$\nabla^2 f(t, z) \Longleftrightarrow -4\pi^2(\mu^2 + \nu^2)F(\mu, \nu).$$

[Hint: Study entry 12 in Table 4.3 and see Problem 4.25(d).]

(b) The preceding closed from expression is valid only for continuous variables. However, it can be the basis for implementing the Laplacian in the discrete frequency domain using $M \times N$ filter $H(u, v) = -4\pi^2(u^2 + v^2)$ for $u = 0, 1, \cdots, M - 1$ and $v = 0, 1, \cdots, N - 1$. Explain how you would implement this filter.

(c) As you saw in Example 4.20, the Laplacian result in the frequency domain was similar to the result of using a spatial mask with a center coefficient equal to $-8$. Explain the reason why the frequency domain result was not similar instead to the result of using a spatial mask with a center coefficient of $-4$. See Section 3.6.2 regarding the Laplacian in the spatial domain.

## 11- Textbook Problem 4.33 (6 points)

Consider the images shown. The image on the right was obtained by: (a) multiplying the image on the left by $(-1)^{x+y}$; (b) computing DFT; (c) taking the complex conjugate of the transform; (d) computing the inverse DFT; and (e) multiplying the real part of the result by $(-1)^{x+1}$. Explain (mathematically) why the image on the right appears as it does. Rotated 180 degrees
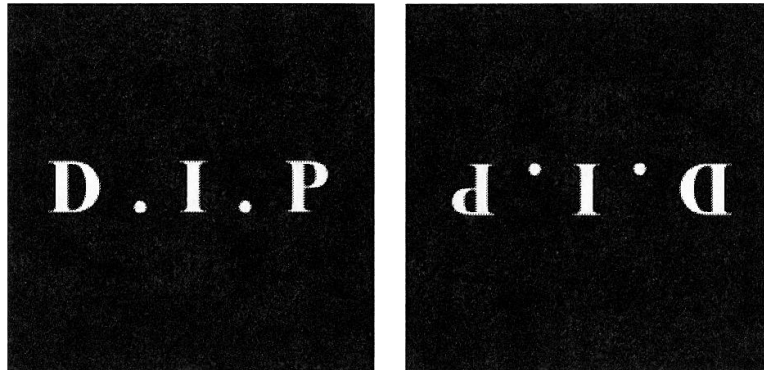
# Programming [30 Points]

For the programming assignments, you can use any programming language or tools. Matlab/Octave are the most suitable tools for the lab assignments in this course since they already have simple and powerful libraries that you can use. If you are using C/C++, Java, Python, or any other programming or scripting language, you may need to find proper libraries or implement them yourself. **If you would like to use any external library outside Matlab/Octave, you should check it with us before using it in your code.**

For this section, you need to submit the code and all the images requested.

## 12- Salt&Pepper Denoising (12 points)

(a) Build a checkerboard image with the following command:

```
a = checkerboard(50, 4, 4);
```

(b) Add salt&pepper noise to the image:

```
an = imnoise(a, 'salt & pepper', d);
```

where `d` is the spatial density of the noise between 0 and 1. Try $d = 1$, $d = 0.1$, and $d = 0.01$.

(c) Apply a median filter on the noisy image to remove the noise. Adjust the filter size to get the best possible image. Explain how the filter works and how the noise density and filter size affects the final image.

(d) Repeat the process for an image of your choice.

## 13- 2D DFT (18 points)

(a) Compute and display the magnitude of the DFT of Figures 4.24(a) and 4.29(a). The Matlab function for the 2D DFT is `fft2`. Use `fftshift` to move the DC component to the center, and use the logarithmic transformation so that your results look like Figures 4.24(d) and 4.29(b), respectively. In both cases, use `imagesc` with CLIM=[5 13] to display the DFT log magnitude.

(b) Compute and display the DFT magnitude (again using the logarithmic transformation) for an image of your own choosing, e.g. taken from the Internet or your own digital camera. If the image is color, use `rgb2gray` to convert it to monochrome.

(c) Apply a perfect low-pass filter with half bandwidth in frequency domain and in spatial domain truncated after 10 samples on each side. Compare the final images.