Tomographic Reconstruction

3D Image Processing

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Reading

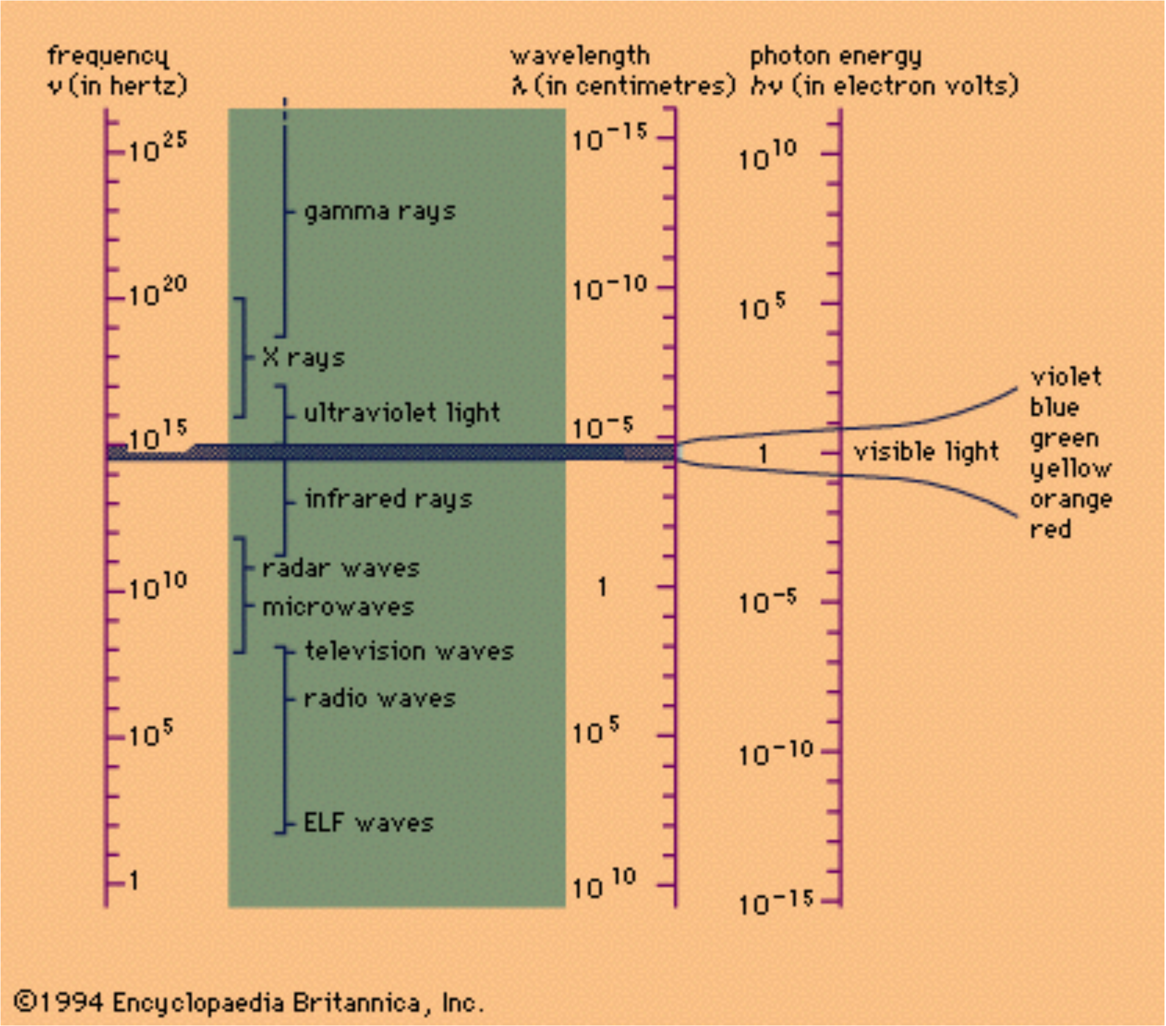
- Gonzales + Woods, Chapter 5.11

Overview

- Physics
- History
- Reconstruction — basic idea
- Radon transform
- Fourier-Slice theorem
- (Parallel-beam) filtered backprojection
- Fan-beam filtered backprojection
- Algebraic reconstruction technique (ART)

X-Rays

- photons produced by an electron beam
- similar to visible light, but higher energy!



X-Rays - Physics

- associated with inner shell electrons
- as the electrons decelerate in the target through interaction, they emit electromagnetic radiation in the form of X-rays.
- patient between an X-ray source and a film -> radiograph
- cheap and relatively easy to use
- potentially damaging to biological tissue

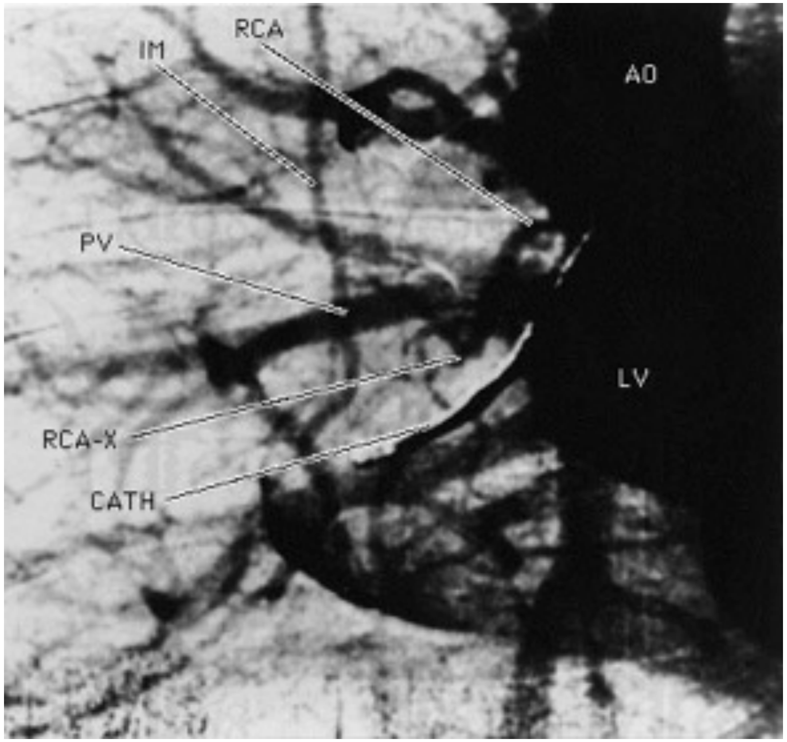
X-Rays - Visibility

- bones contain heavy atoms -> with many electrons, which act as an absorber of X-rays
- commonly used to image gross bone structure and lungs
- excellent for detecting foreign metal objects
- main disadvantage -> lack of anatomical structure
- all other tissue has very similar absorption coefficient for X-rays

X-Rays - Angiography

- inject contrast medium (electron dense dye)
- used to image vasculature (blood vessels)
- angiocardiography (heart)
- cholecystography (gall bladder)
- myelography (spinal cord)
- urography (urinary tract)

X-Rays - Images

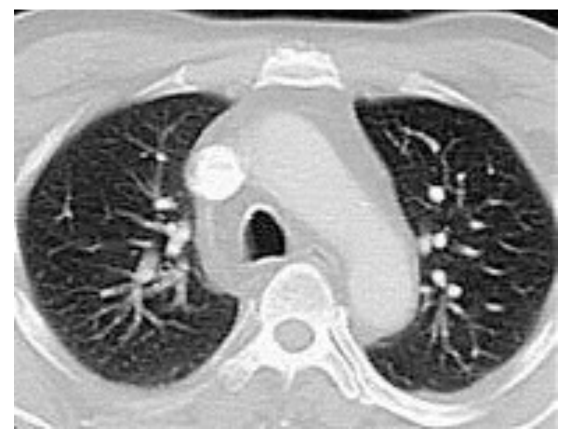


CT or CAT - Principles

- Computerized (Axial) Tomography
- introduced in 1963/1972 by Hounsfield and Cormack (1979 Noble prize in medicine)
- natural progression from X-rays
- based on the principle that a three-dimensional object can be reconstructed from its two dimensional projections
- based on the Radon transform (a map from an n-dimensional space to an (n-1)-dimensional space)

CT or CAT - Methods

- measures the attenuation of X-rays from many different angles
- a computer reconstructs the organ under study in a series of cross sections or planes
- combine X-ray pictures from various angles to reconstruct 3D structures



video

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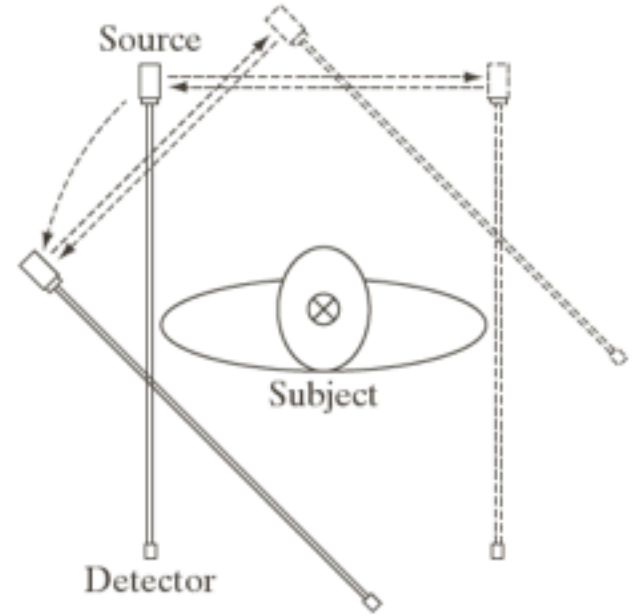
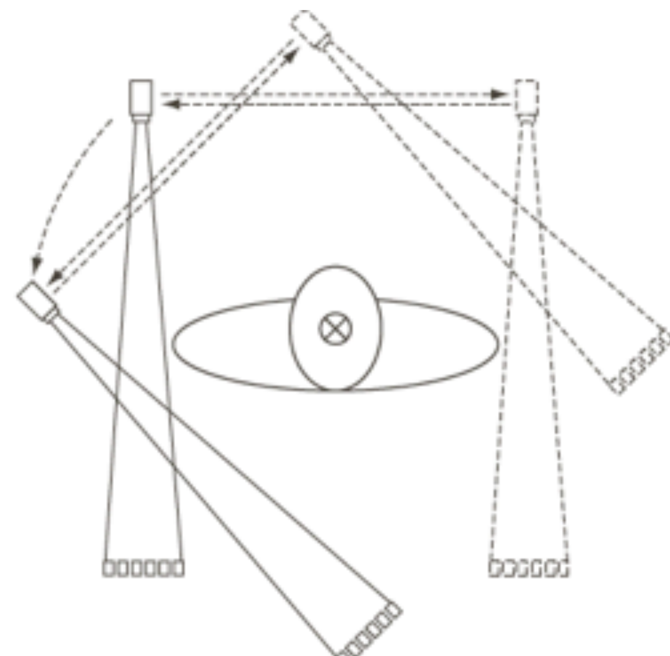
History

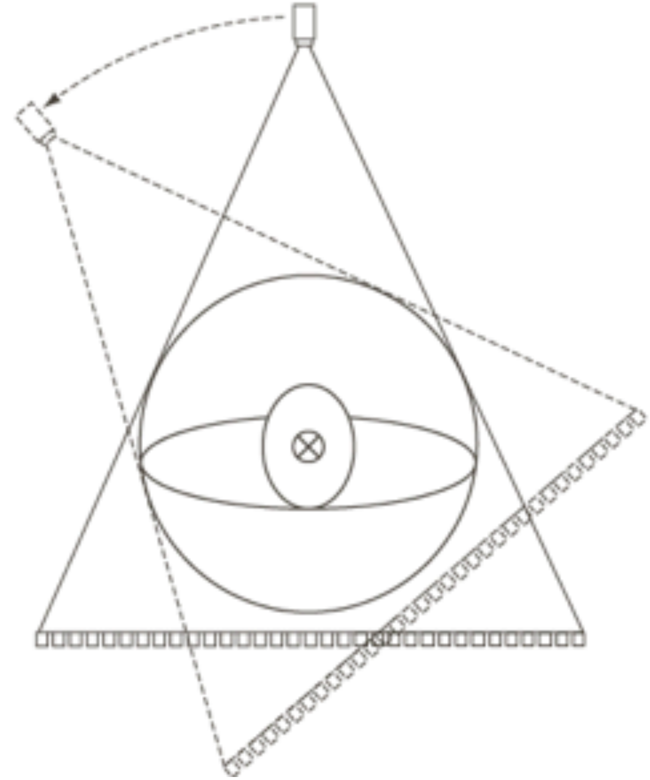
- G1 (first gen) CT:
  - employ “pencil” X-ray beam
  - single detector
  - linear translation of source/detector pair
- G2 CT:
  - same as G1, but
  - beam is shaped as a fan
  - allows multiple detectors

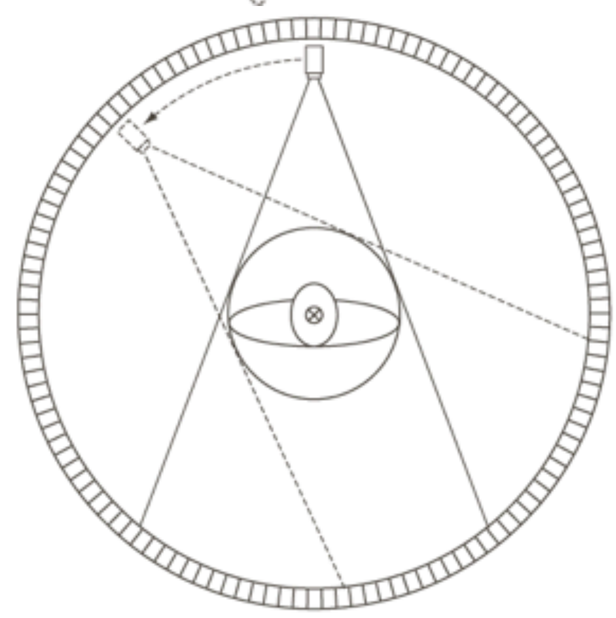




History

- G3 CT:
  - great improvement with bank of detectors (~1000 detectors)
  - no need for translation
- G4 CT:
  - circular ring of detectors (~5000 detectors)
- G3+G4:
  - higher speed
  - higher dose and higher cost





Modern scanners

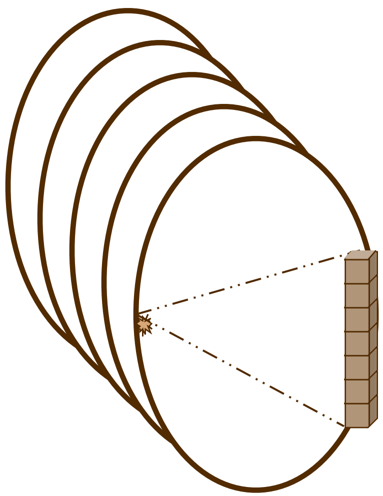
- G5 CT (EBCT — electron beam CT)
  - eliminate mechanical motion
  - beams controlled electromagnetically
- G1-G5: one image at a time, then patient is moved.
- Patient must hold breath for lung CT

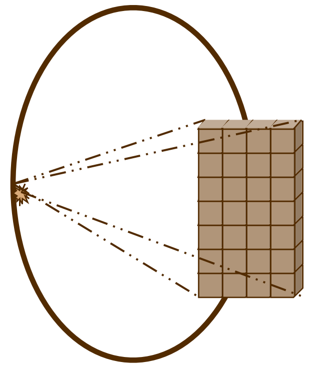
Modern scanners

- G6 CT (helical CT):
  - continuous movement of patient and source/detector
- G7 CT (multi-slice CT):
  - thick fan-beams, collecting multiple slices at once
  - reducing cost + dosage

CT - 2D vs. 3D

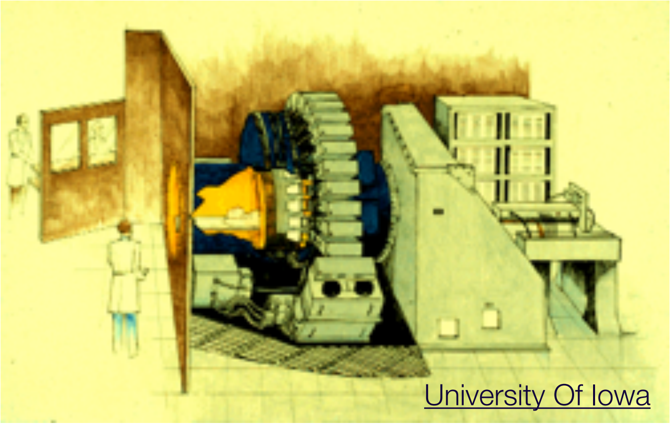
- Linear advancement (slice by slice)
  - typical method
  - tumor might fall between ‘cracks’
  - takes long time
- helical movement
  - 5-8 times faster
  - under-utilization of cone beam
  - heart synchronization difficult
- 2D projections
  - enhanced speed

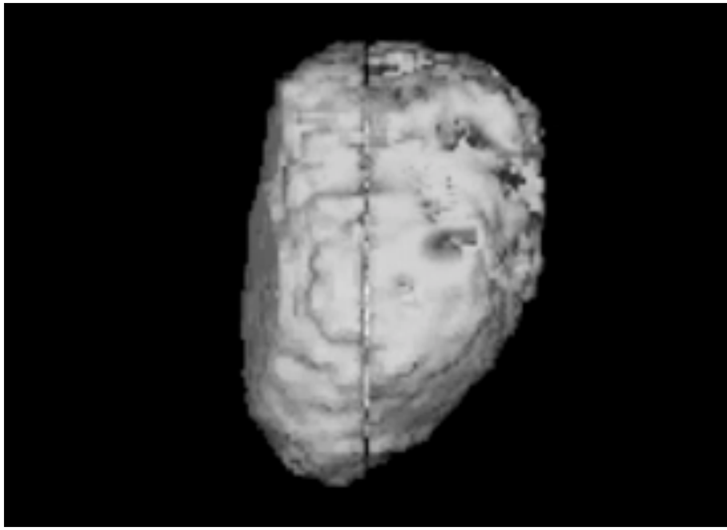




CT - Beating Heart?

- Noise if body parts move!
- Heart - synchronize imaging with heart beat
  - can't capture beating well
  - need faster techniques
- Dynamic Spatial Reconstructor
  - has 14 X-ray/camera pairs
  - but turns slower
  - 2D projections seem more plausible
  - and cheaper





CT or CAT - Advantages

- significantly more data is collected
- superior to single X-ray scans
- far easier to separate soft tissues other than bone from one another (e.g. liver, kidney)
- data exist in digital form -> can be analyzed quantitatively
- adds enormously to the diagnostic information
- used in many large hospitals and medical centers throughout the world

CT or CAT - Disadvantages

- significantly more data is collected
- soft tissue X-ray absorption still relatively similar
- still a health risk
- MRI is used for a detailed imaging of anatomy

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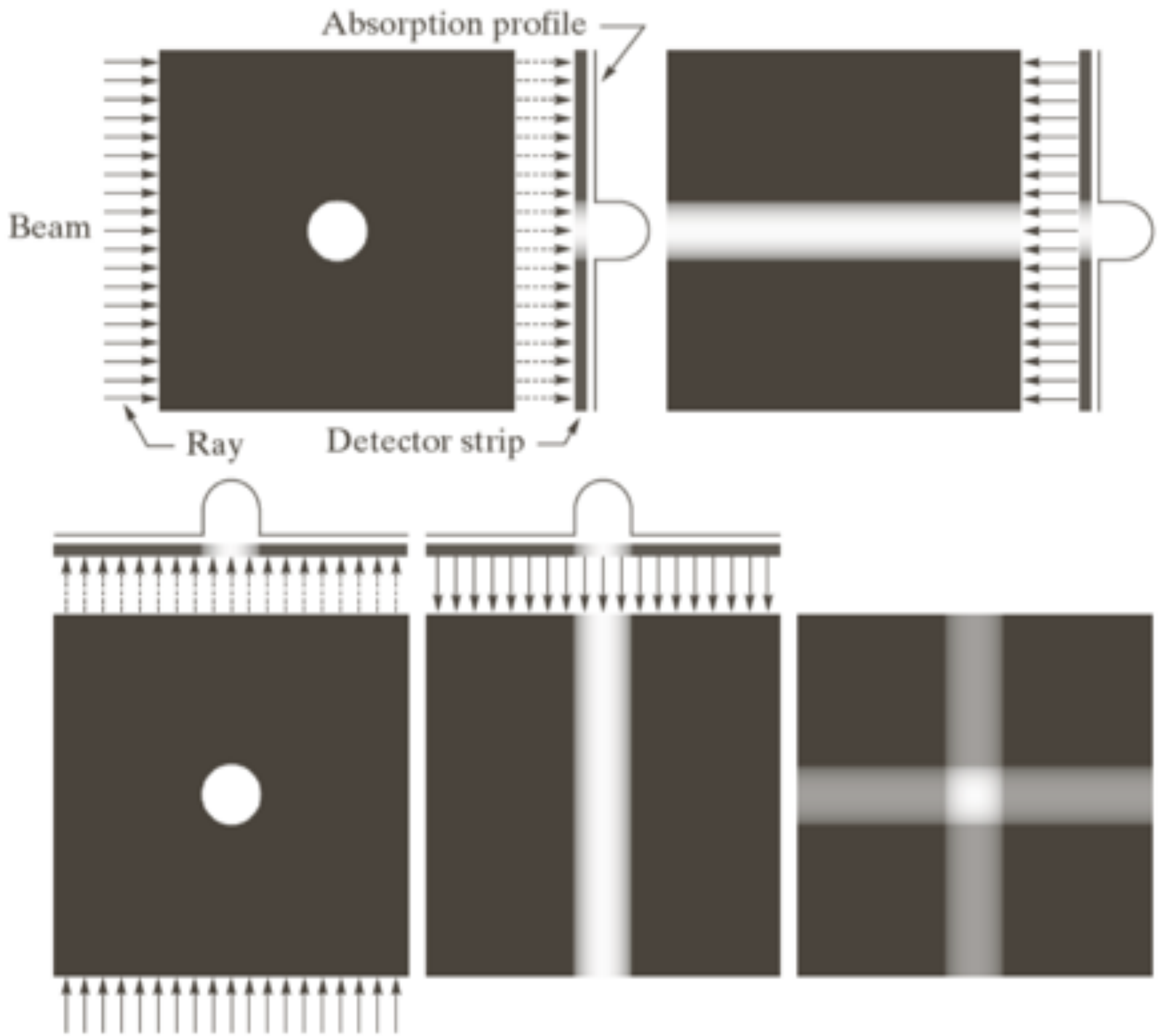
Basics: Projection vs. backprojection

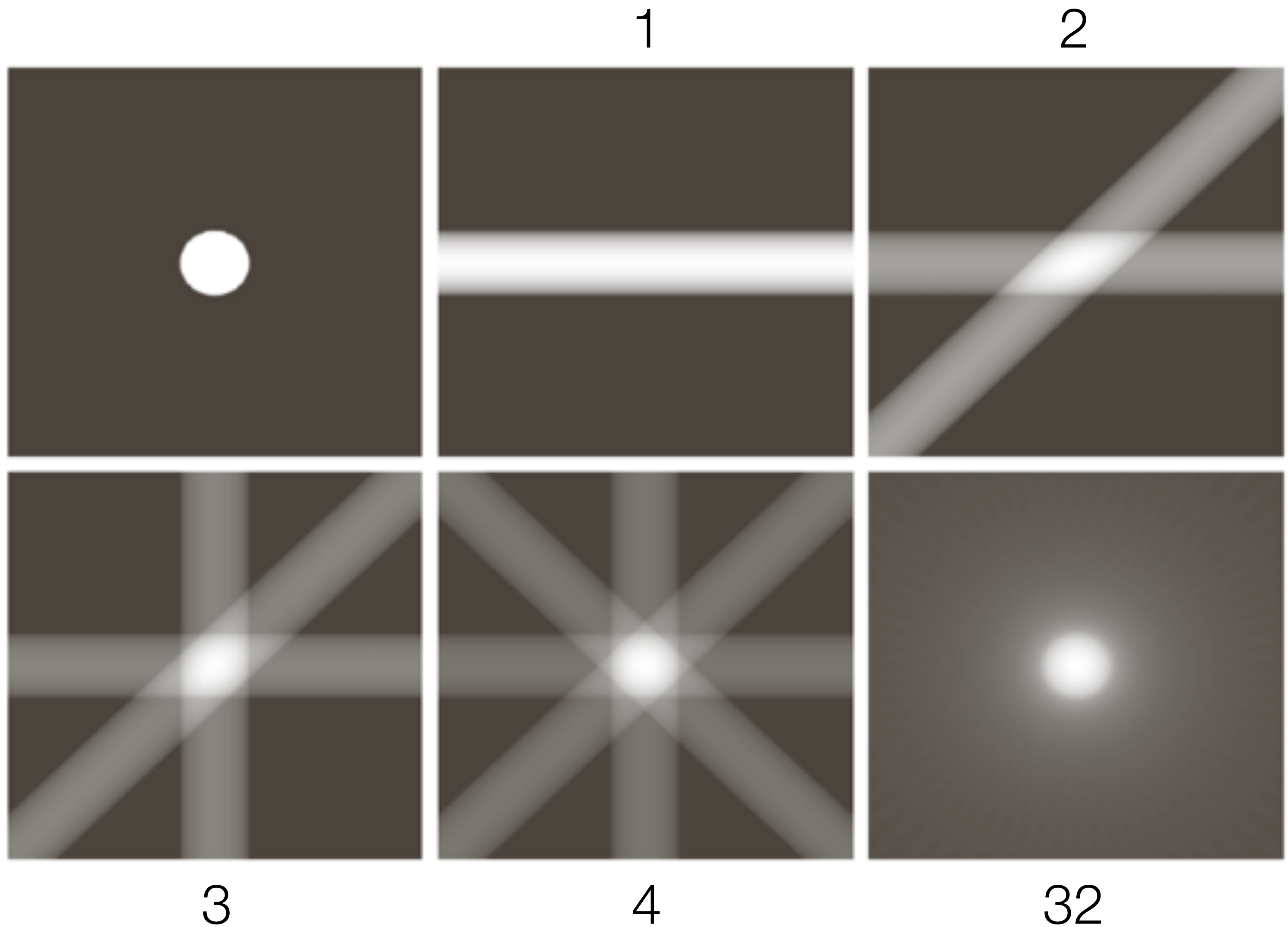
Projection



Back-projection

a	b	c
d	e	f

FIGURE 5.33
(a) Same as Fig. 5.32(a).
(b)–(e) Reconstruction using 1, 2, 3, and 4 backprojections 45° apart.
(f) Reconstruction with 32 backprojections 5.625° apart (note the blurring).



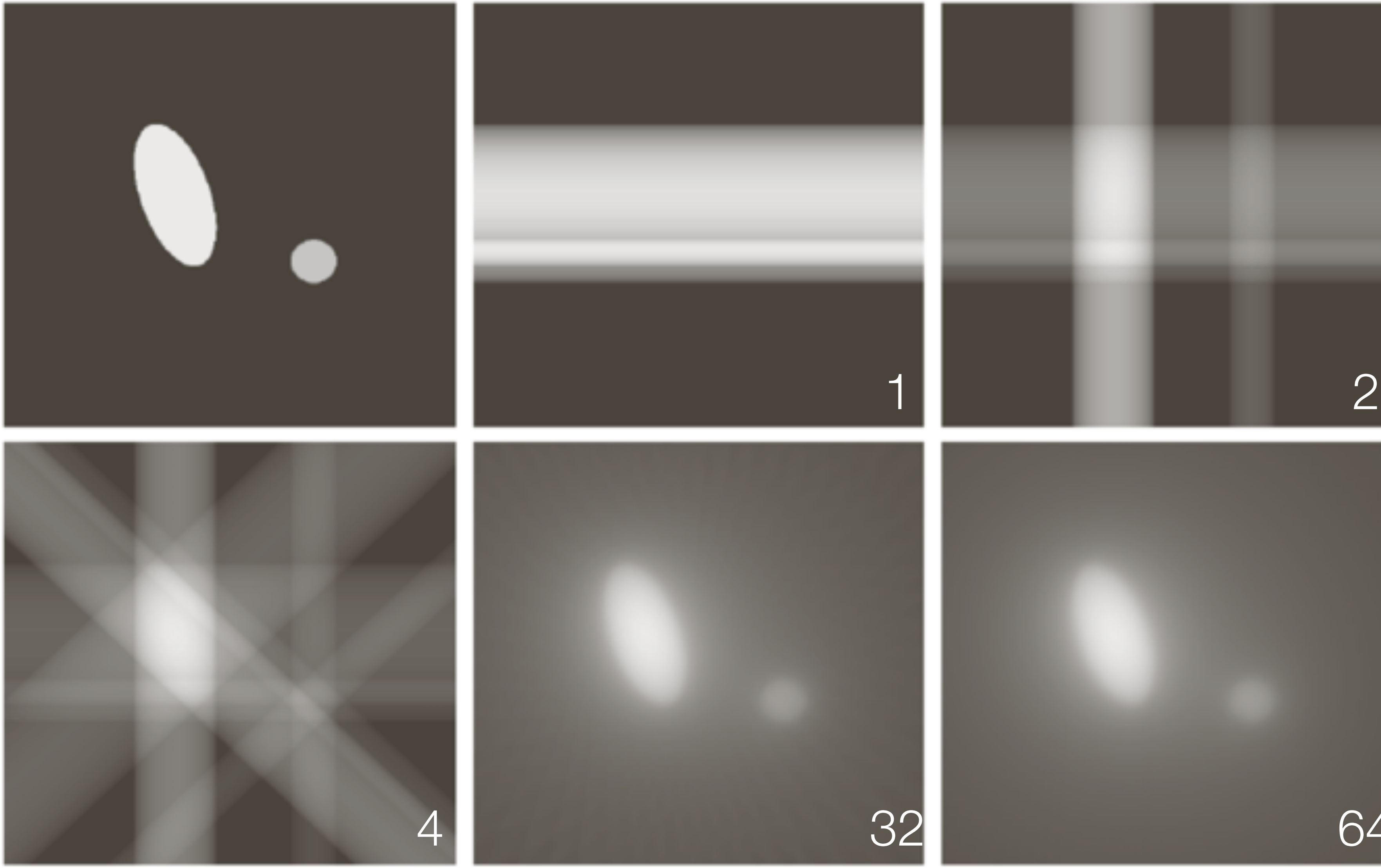


FIGURE 5.34 (a) A region with two objects. (b)–(d) Reconstruction using 1, 2, and 4 backprojections 45° apart. (e) Reconstruction with 32 backprojections 5.625° apart. (f) Reconstruction with 64 backprojections 2.8125° apart.

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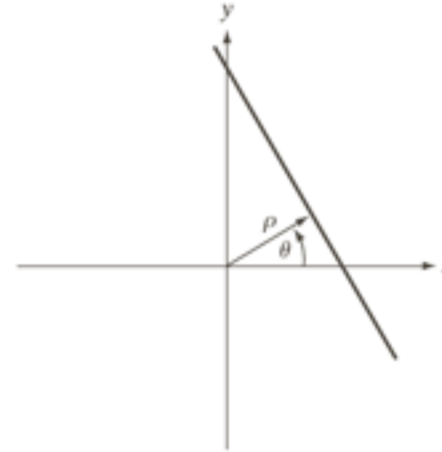
Math. principles



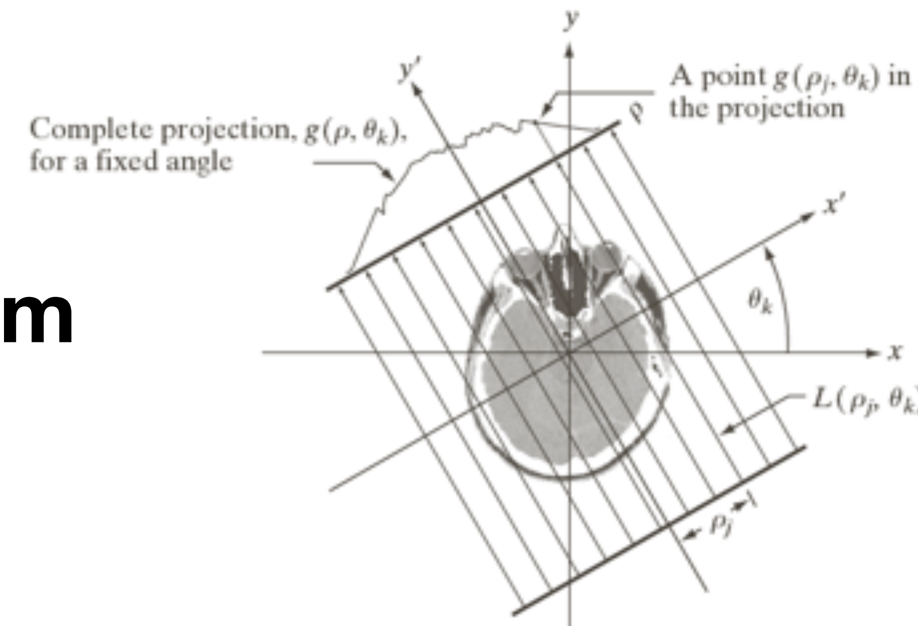
- Expression of a simple line:

$$x\cos\theta + y\sin\theta = \rho$$

- computing projections:

$$g(\rho_j,\theta_k) = \int\int f(x,y)\delta(x\cos\theta_k + y\sin\theta_k - \rho_j)dxdy$$

- Also known as **Radon Transform**



Sinogram

- Radon transform written as simple images

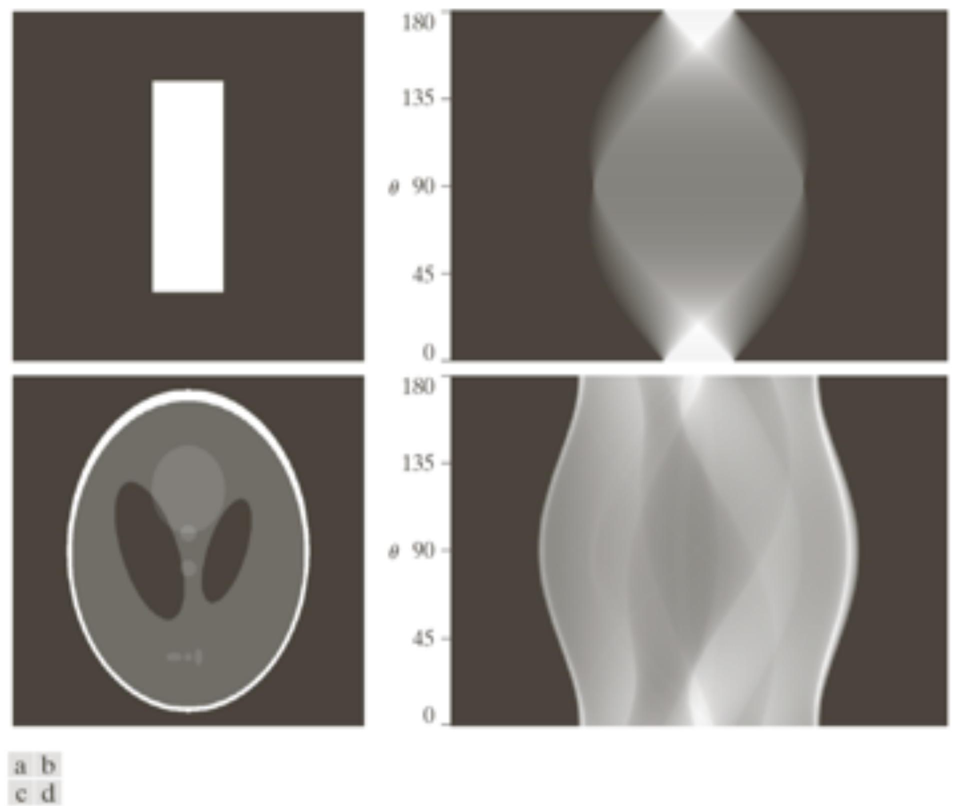


FIGURE 5.39 Two images and their sinograms (Radon transforms). Each row of a sinogram is a projection along the corresponding angle on the vertical axis. Image (c) is called the *Shepp-Logan phantom*. In its original form, the contrast of the phantom is quite low. It is shown enhanced here to facilitate viewing.

Simple reconstruction

- mathematically "smearing" is:

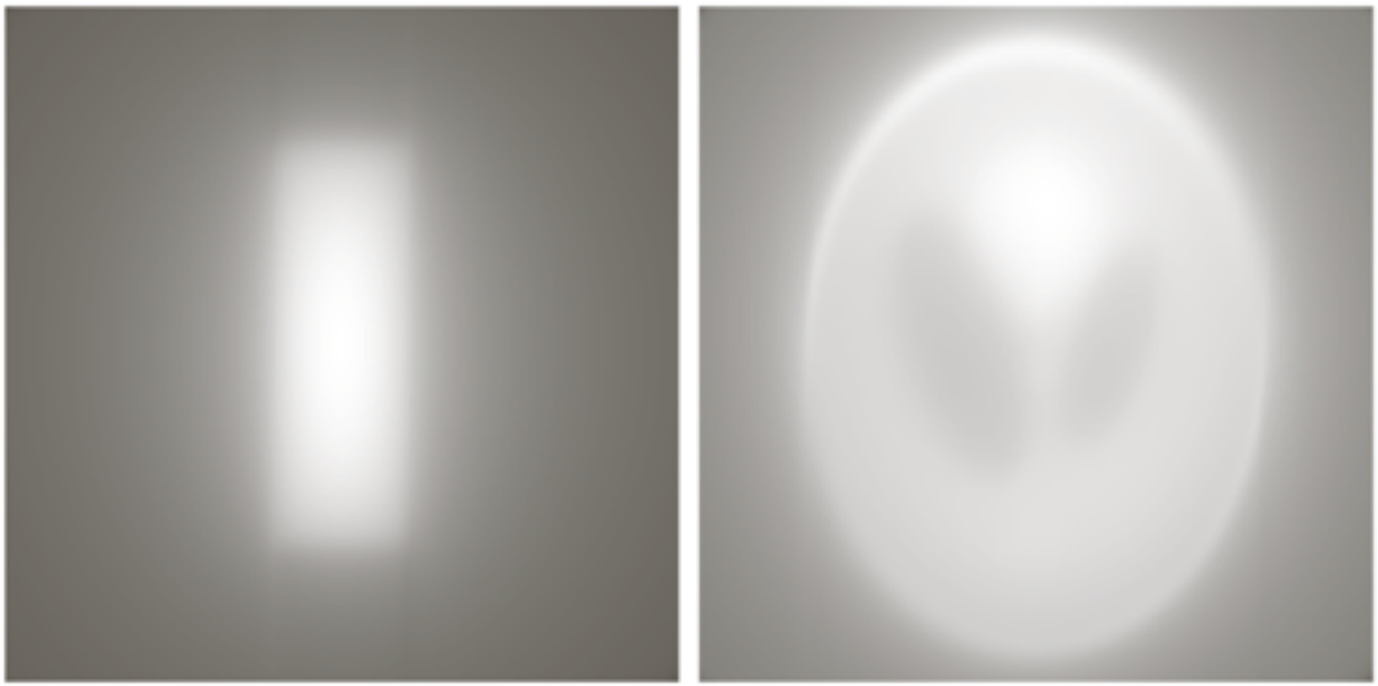
$$f_{\theta_k}(x,y) = g(\rho,\theta_k)$$
$$= g(x\cos\theta_k + y\sin\theta_k, \theta_k)$$

- simply summing it all up:

$$f(x,y) = \int_0^{\pi} f_\theta(x,y)d\theta$$

Simple reconstruction

- leads to lots of blurring



- proper reconstruction — projection slice

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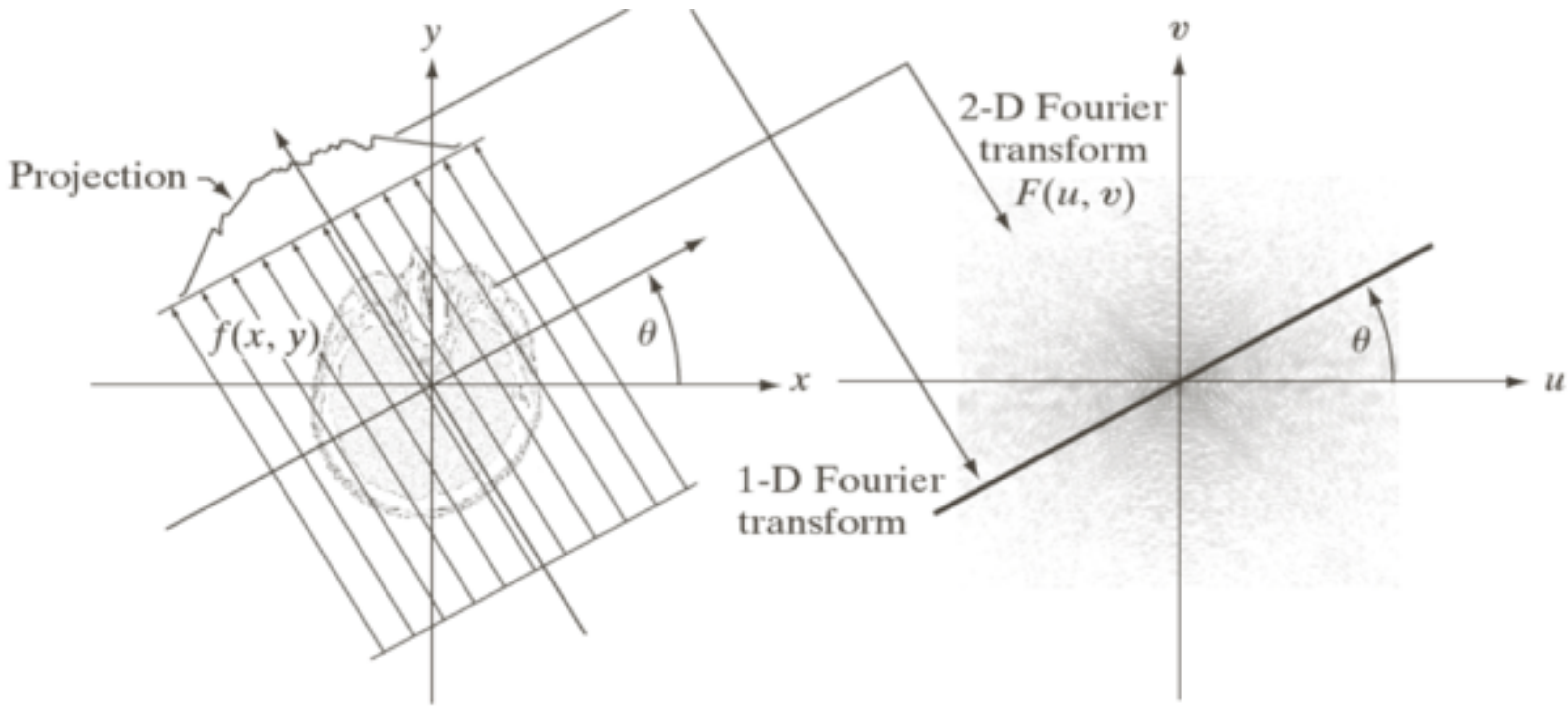
Projection-slice theorem



$$G(\omega, \theta) = \int_{-\infty}^{\infty} g(\rho, \theta) e^{-j2\pi\omega\rho} d\rho$$

Projection-slice theorem

- mathematically

$$G(\omega,\theta) = \int\int\int f(x,y)\delta(x\cos\theta + y\sin\theta - \rho)e^{-j2\pi\omega\rho}dxdyd\rho$$

$$= \int\int f(x,y)\left[\int \delta(x\cos\theta + y\sin\theta - \rho)e^{-j2\pi\omega\rho}d\rho\right]dxdy$$

$$= \int\int f(x,y)e^{-j2\pi\omega(x\cos\theta + y\sin\theta)}dxdy$$

- meaning:

$$G(\omega,\theta) = \left[\int\int f(x,y)e^{-j2\pi(ux+vy)}dxdy\right]_{u=\omega\cos\theta; v=\omega\sin\theta}$$

$$= F(\omega\cos\theta, \omega\sin\theta)$$

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some math

$$dudv = \omega d\omega d\theta$$

$$f(x,y) = \int\int F(x,y)e^{j2\pi(ux+vy)}dudv$$

$$G(\omega, \theta + \pi) = G(-\omega, \theta)$$

some math

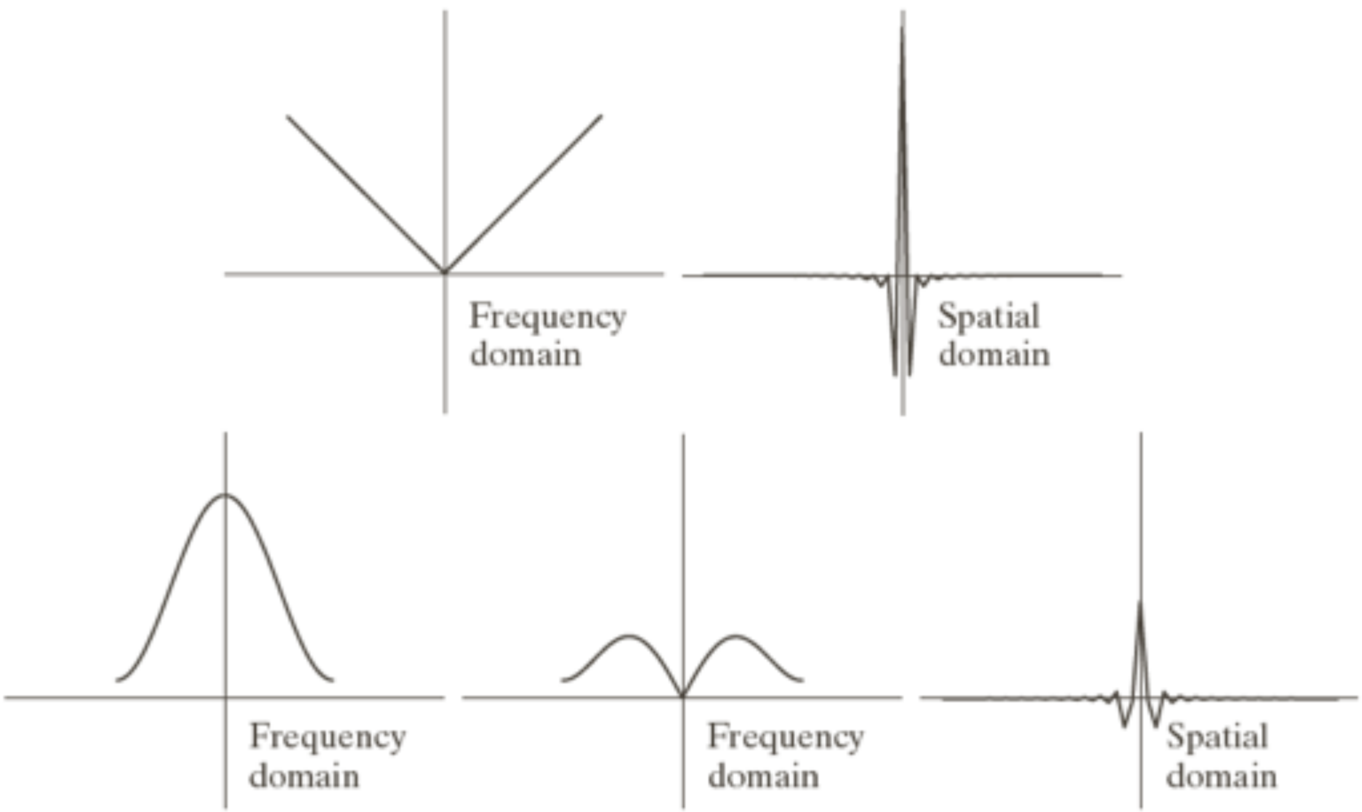
$$dudv = \omega d\omega d\theta$$

$$G(\omega, \theta + \pi) = G(-\omega, \theta)$$

$$\begin{aligned}
f(x,y) &= \int\int F(x,y) e^{j2\pi(ux+vy)} dudv \\
&= \int_0^{2\pi}\int_0^{\infty} F(\omega\cos\theta, \omega\sin\theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)} \omega d\omega d\theta \\
&= \int_0^{2\pi}\int_0^{\infty} G(\omega, \theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)} \omega d\omega d\theta \\
&= \int_0^{\pi}\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)} d\omega d\theta \\
&= \int_0^{\pi} \left[\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega\rho} d\omega\right]_{\rho = x\cos\theta + y\sin\theta} d\theta
\end{aligned}$$

Multiplication with a ramp

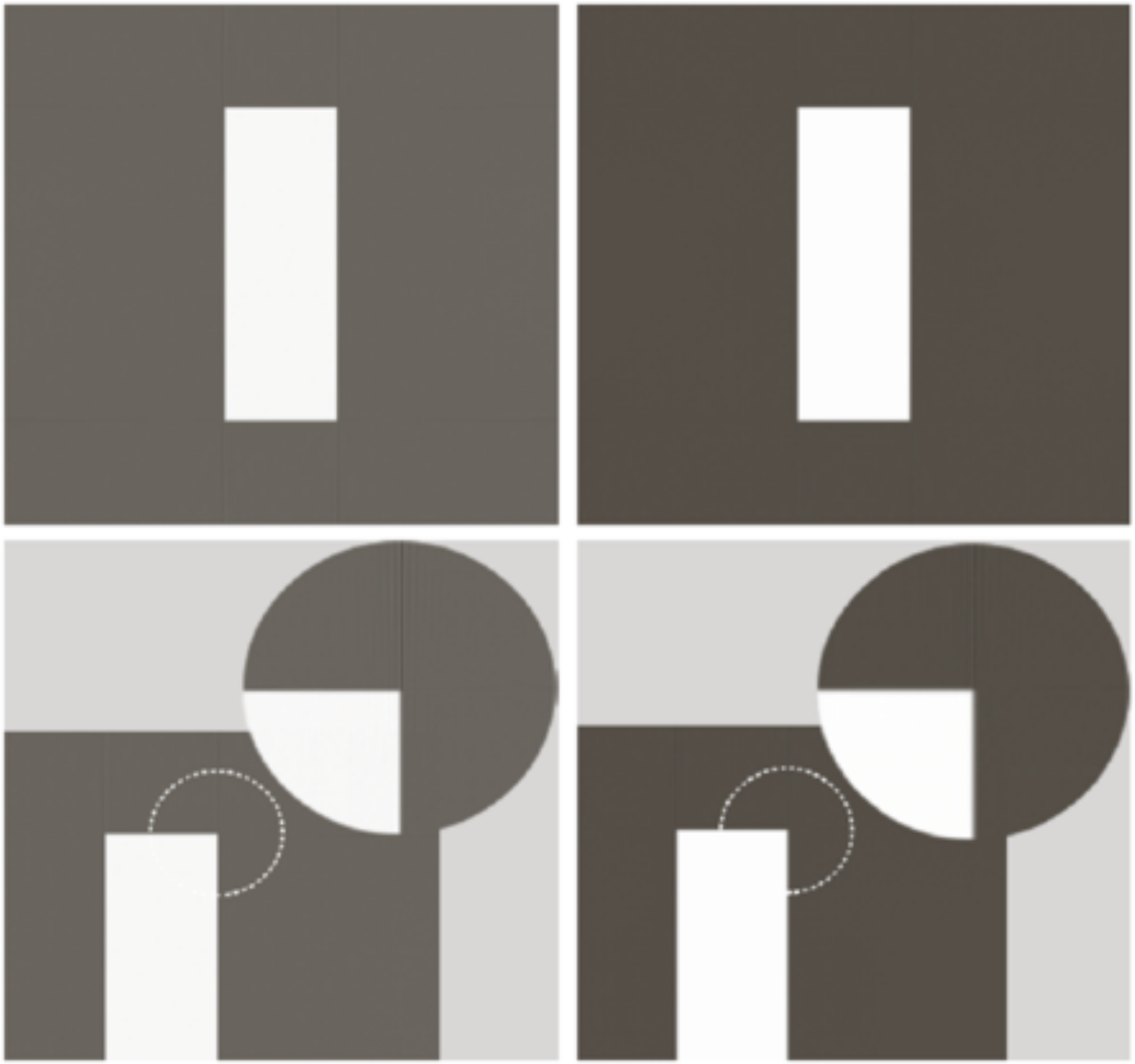
- sharp cut-off, yields ringing!



a b
c d e

FIGURE 5.42
(a) Frequency domain plot of the filter $|\omega|$ after band-limiting it with a box filter. (b) Spatial domain representation. (c) Hamming windowing function. (d) Windowed ramp filter, formed as the product of (a) and (c). (e) Spatial representation of the product (note the decrease in ringing).

Multiplication with a ramp



a	b
c	d

FIGURE 5.43 Filtered back-projections of the rectangle using (a) a ramp filter, and (b) a Hamming-windowed ramp filter. The second row shows zoomed details of the images in the first row. Compare with Fig. 5.40(a).

Multiplication with a ramp

a b

FIGURE 5.44 Filtered backprojections of the head phantom using (a) a ramp filter, and (b) a Hamming-windowed ramp filter. Compare with Fig. 5.40(b).

Filtered Backpropagation

$$dudv = \omega d\omega d\theta$$

$$G(\omega, \theta + \pi) = G(-\omega, \theta)$$

$$
\begin{aligned}
f(x,y) &= \int\int F(x,y)e^{j2\pi(ux+vy)}dudv \\
&= \int_0^{2\pi}\int_0^{\infty} F(\omega\cos\theta, \omega\sin\theta)e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta \\
&= \int_0^{2\pi}\int_0^{\infty} G(\omega,\theta)e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta \\
&= \int_0^{\pi}\int_{-\infty}^{\infty} |\omega|G(\omega,\theta)e^{j2\pi\omega(x\cos\theta+y\sin\theta)}d\omega d\theta \\
&= \int_0^{\pi}\left[\int_{-\infty}^{\infty} |\omega|G(\omega,\theta)e^{j2\pi\omega\rho}d\omega\right]_{\rho=x\cos\theta+y\sin\theta} d\theta
\end{aligned}
$$

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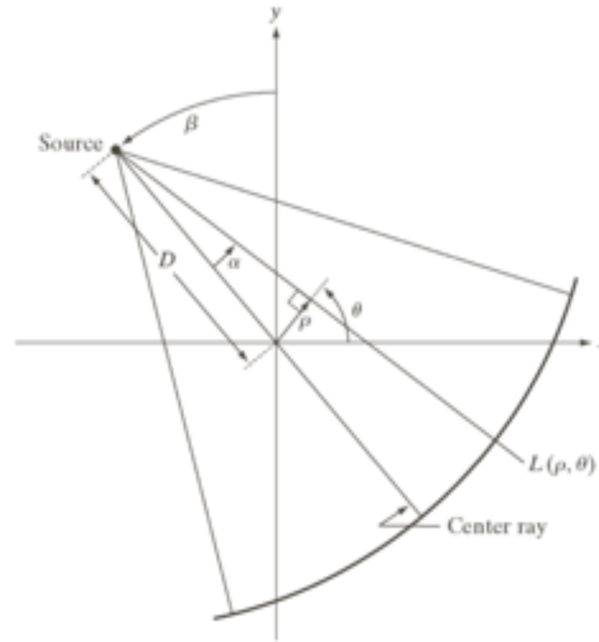
Principle idea

- simple: just untangle all the beams from the fan into the right parallel beam reconstruction
- more direct: $$\theta = \alpha + \beta$$ $$\rho = D \sin \alpha$$



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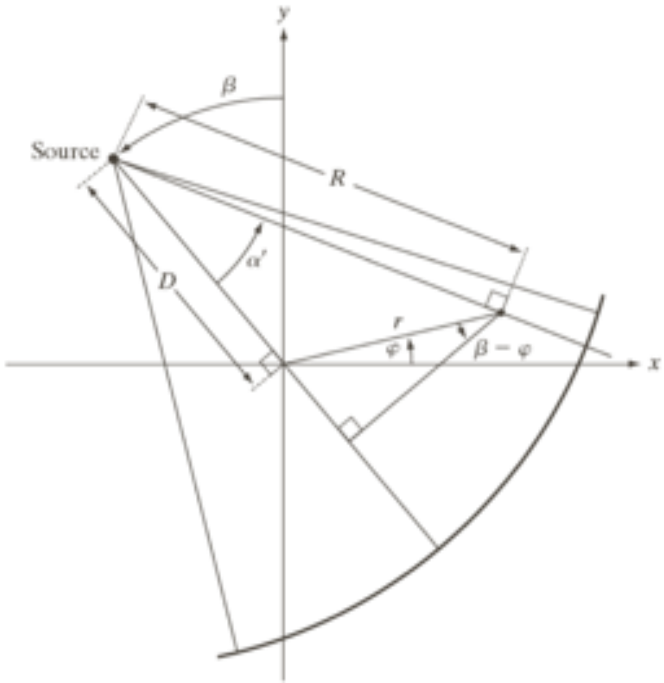
- more direct: $\theta = \alpha + \beta$

$$\rho = D \sin \alpha$$



$$f(r, \phi) = \int_0^{2\pi} \frac{1}{R^2} \left[\int_{-\alpha_m}^{\alpha_m} q(\alpha, \beta) h(\alpha' - \alpha) d\alpha \right] d\beta$$

$$h(\alpha) = \frac{1}{2} \left(\frac{\alpha}{\sin \alpha} \right)^2 s(\alpha)$$

Results

- typically need more projections!

a	b
c	d

FIGURE 5.48 Reconstruction of the rectangle image from filtered fan backprojections. (a) 1° increments of α and β. (b) 0.5° increments. (c) 0.25° increments. (d) 0.125° increments. Compare (d) with Fig. 5.43(b).

Results

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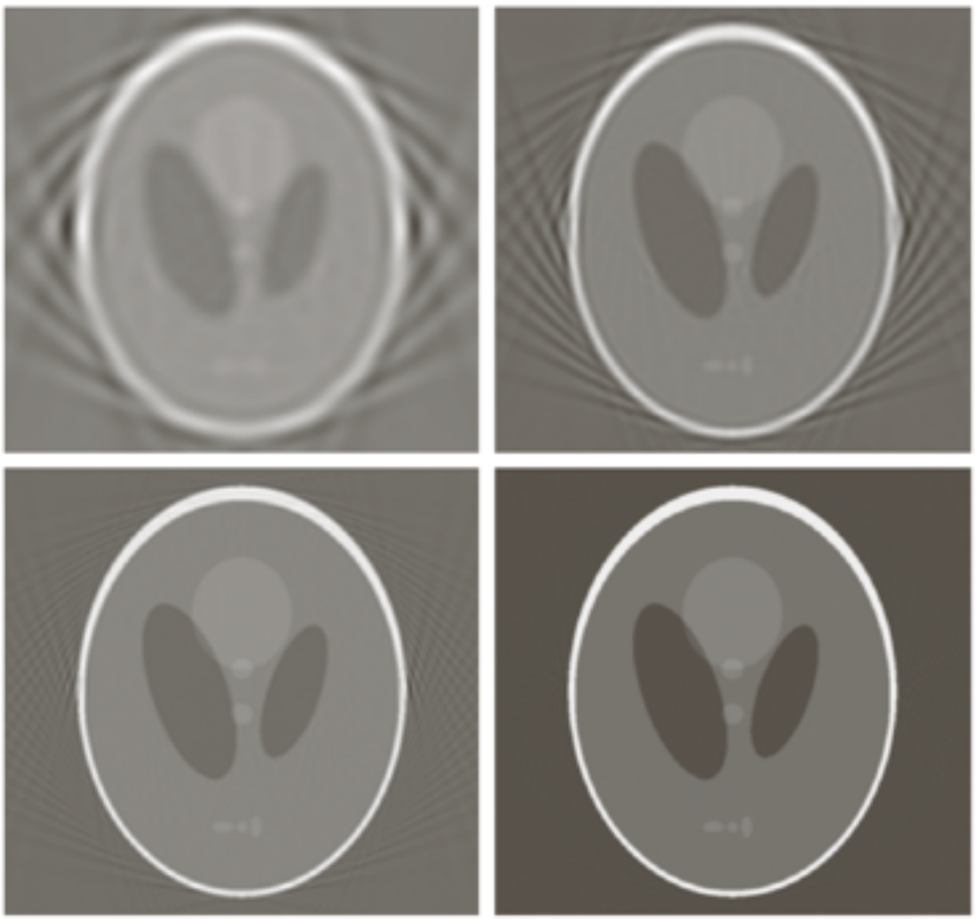

a b
c d

FIGURE 5.49 Reconstruction of the head phantom image from filtered fan backprojections. (a) 1° increments of α and β. (b) 0.5° increments. (c) 0.25° increments. (d) 0.125° increments. Compare (d) with Fig. 5.44(b).

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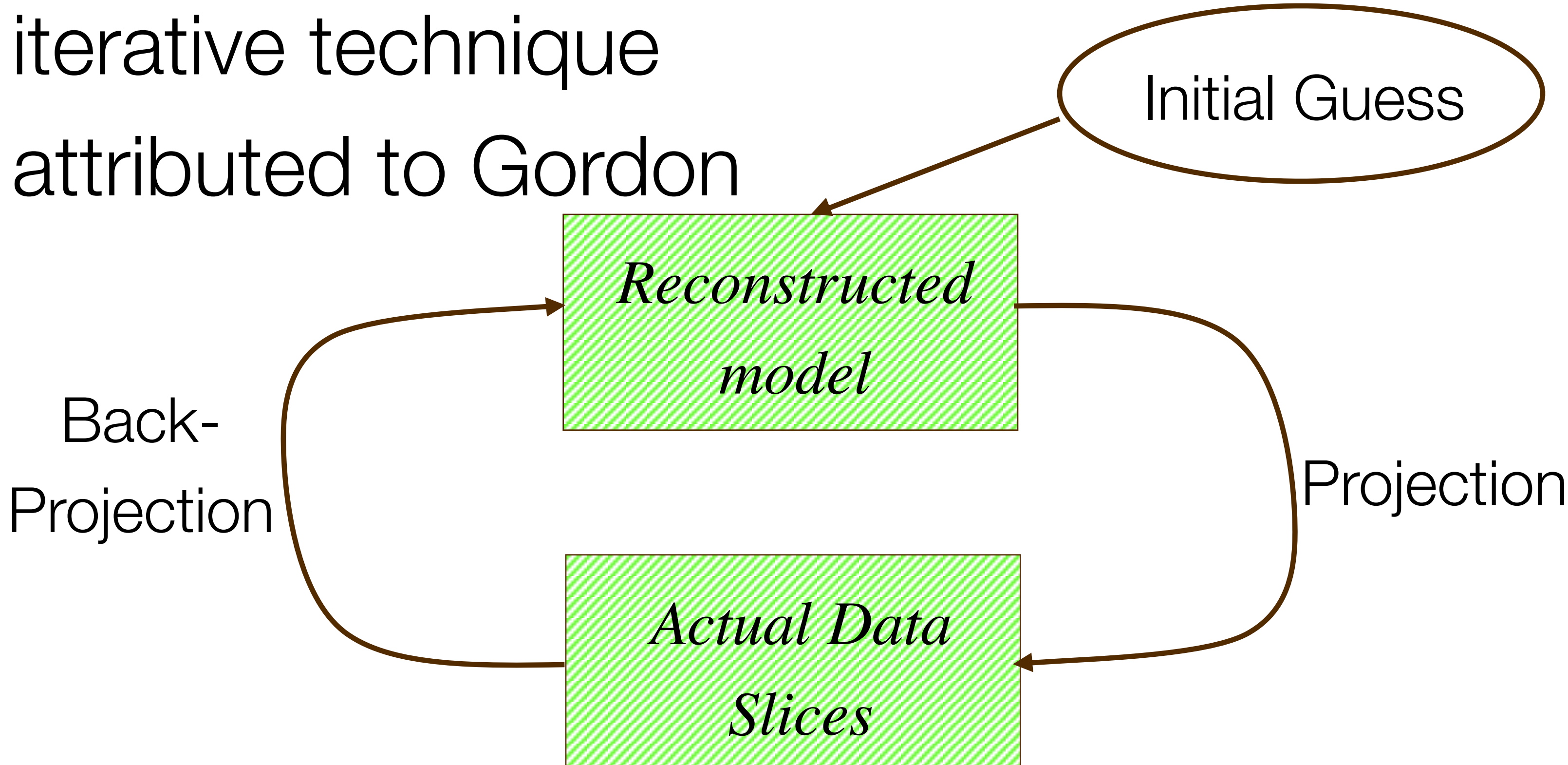
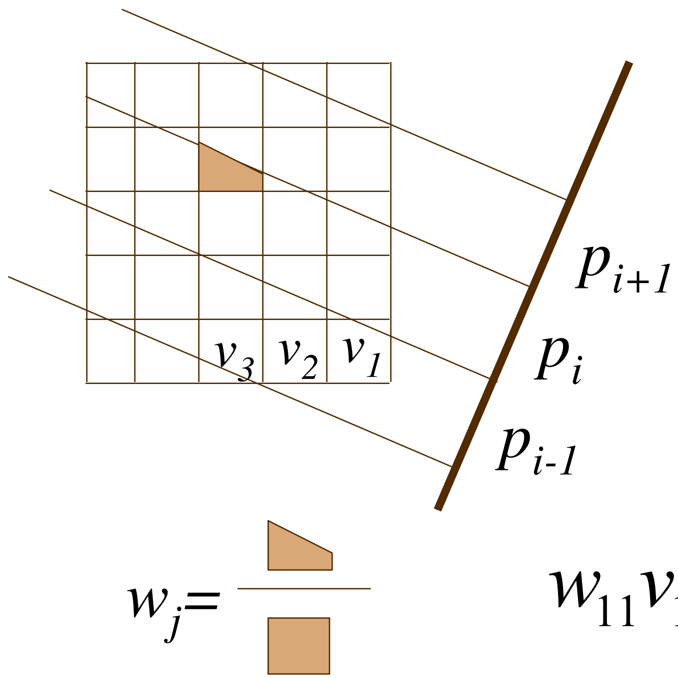
CT - Reconstruction: ART/EM

- Algebraic Reconstruction Technique
- Expectation Maximization (EM)
- iterative technique
- attributed to Gordon



CT - Reconstruction: ART (2)



$$w_{11}v_1 + w_{12}v_2 + \ldots + w_{1N}v_N = p_1$$
$$w_{21}v_1 + w_{22}v_2 + \ldots + w_{2N}v_N = p_2$$
$$\vdots$$
$$w_{M1}v_1 + w_{M2}v_2 + \ldots + w_{MN}v_N = p_M$$

CT - Reconstruction: ART (3)

- object reconstructed on a discrete grid by a sequence of alternating grid projections and correction back-projections.
- Projection: measures how close the current state of the reconstructed object matches one of the scanner projections
- Back-projection: corrective factor is distributed back onto the grid
- many projection/back-projection steps needed for a certain tolerance margin

CT - FBP vs. ART

FBP

- Computationally cheap
- Clinically usually 500 projections per slice
- problematic for noisy projections

ART

- Still slow
- better quality for fewer projections
- better quality for non-uniform project.
- “guided” reconstruct. (initial guess!)