3D Viewing

Introduction to Computer Graphics
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Reading

- Chapter 5 of Angel
- Chapter 13 of Hughes, van Dam, ...
- Chapter 7 of Shirley+Marschner
Objectives

- What kind of camera we use? (pinhole)
- What projections make sense
  - Orthographic
  - Perspective
- The viewing pipeline
- Viewing in WebGL
- Shadows
3D Viewing

- Popular analogy: virtual camera taking pictures in a virtual world
- The process of getting an image onto the computer screen is like that of taking a snapshot.
3D Viewing (2)

- With a camera, one:
  - establishes the view
  - opens the shutter and exposes the film
3D Viewing (3)

• With a camera, one:
  – establishes the view
  – opens the shutter and exposes the film

• With a computer, one:
  – chooses a projection type (not necessarily perspective)
  – establishes the view
  – clips the scene according to the view
  – projects the scene onto the computer display
Normal Camera Lens

http://michaeldmann.net/pix_7/lenses.gif
The depth of field will increase to infinity. For

The ideal pinhole camera

- Single ray of light gets through small pinhole
- Film placed on side of box opposite to pinhole

\[ y_p = -\frac{y}{z/d} \]

\[ x_p = -\frac{x}{z/d} \]

Projection
The pinhole camera

• Angle of view (always fixed)

\[ \theta = 2 \tan^{-1} \frac{h}{2d} \]

• Depth of field (DOF): infinite – every point within the field of view is in focus (the image of a point is a point)

• Problem: just a single ray of light & fixed view angle

• Solution: pinhole \( \rightarrow \) lens, DOF no longer infinite
The synthetic camera model

- Image formed **in front of** the camera
- **Center of projection (COP)**
  center of the lens (eye)
Types of projections

• Choose an appropriate type of projection (not necessarily perspective)
• Establishes the view: direction and orientation
3D viewing with a computer

- **Clips scene** with respect to a view volume
  - Usually finite to avoid extraneous objects

- **Projects the scene** onto a projection plane
  - In a similar way as for the synthetic camera model
  - Everything in view volume is in focus
  - Depth-of-field (blurring) effects may be generated
Where are we at?

Clip against 3D view volume

Project onto projection plane

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Projection: 3D → 2D

• We study planar geometric projections:
  – projecting onto a flat 2D surface
  – project using straight lines

• Projection rays, called *projectors*, are sent through each point in the scene from the centre of projection (COP) - our pinhole

• Intersection between projectors and projection plane form the projection
Perspective and parallel projections

- **Perspective:**
  - Determined by COP

- **Parallel:**
  - COP at infinity
  - By direction of projectors (DOP)
COP in homogeneous coordinates

- Perspective projection: COP is a finite point: \((x, y, z, 1)\)
- Parallel projection
  - Direction of projection is a vector:
    \((x, y, z, 1) - (x', y', z', 1) = (a, b, c, 0)\)
  - Points at infinity and directions correspond in a natural way
Perspective vs. parallel

• Perspective projection:
  – Realistic, mimicking our human visual system
  – **Foreshortening**: size of perspective projection of object varies inversely with the distance of that object from the center of projection
  – Distances, angles, parallel lines are not preserved

• Parallel projection:
  – Less realistic but can be used for measurements
  – Foreshortening uniform, i.e., not related to distance
  – Parallel lines are preserved (length preserving?)
Taxonomy of projections

Angle between projectors and projection plane?

Number of principal axes cut by projection plane

Parallel
- Orthographic
  - Top (plan)
  - Front elevation
- Axonometric
  - Side elevation
- Isometric
  - Other

Oblique
- Cabinet
- Cavalier
- Isometric
  - Other

One-point
- Perspective
  - Two-point
  - Three-point

Planar geometric projections
Parallel projections

- Used in engineering mostly, e.g., architecture
- Allow measurements
- Different uniform foreshortenings are possible, i.e., not related to distance to projection plane
- Parallel lines remain parallel
- Angles are preserved only on faces which are parallel to the projection plane – same with perspective projection
Orthographic (parallel) projections

- Projectors are **normal** to the projection plane
- Commonly front, top (plan) and side elevations: projection plane perpendicular to z, y, and x axis
- Matrix representation (looking towards negative z along the z axis; projection plane at z = 0):

\[ M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \]
Orthographic projections

- **Axonometric projection**
  - Projection plane not normal to any principal axis
  - E.g., can see more faces of an axis-aligned cube
Orthographic projections

- Foreshortening: three scale factors, one each for x, y, and z axis
- Axonometric projection example:

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos\alpha & -\sin\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix} \times \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos\alpha & -\sin\alpha & 0 & 0 \\
0 & 0 & \sin\alpha & \cos\alpha & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
Orthographic projection

- Axonometric projection – **Isometric**
  - Projection-plane normal makes same angle with each principal axis
  - There are just eight normal directions of this kind
  - All three principal axes are equally foreshortened, good for getting measurements
  - Principal axes make same angle in projection plane

- Alternative: **dimetric** & **trimetric** (general case)
Parallel Projection (4)

- Axonometric Projection - Isometric
  - Angles between the projection of the axes are equal i.e. 120°

- Alternative
  - dimetric & trimetric
Types of Axonometric Projections

Dimetric  Trimetric  Isometric
Oblique (parallel) projections

- Projectors are not normal to projection plane

- Most drawings in the text use oblique projection
Oblique projections

- Two angles are of interest:
  - Angle $\alpha$ between the projector and projection plane
  - The angle $\phi$ in the projection plane
- Derive the projection matrix
Derivation of oblique projections

\[ x_p = x + L \cos \phi \]
\[ y_p = y + L \sin \phi \]

\[ L = \frac{z}{\tan \alpha} \]

\[
M = \begin{bmatrix}
1 & 0 & \cos \phi / \tan \alpha & 0 \\
0 & 1 & \sin \phi / \tan \alpha & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Common oblique projections

- **Cavalier projections**
  - Angle $\alpha = 45$ degrees
  - Preserves the length of a line segment perpendicular to the projection plane
  - Angle $\phi$ is typically 30 or 45 degrees

- **Cabinet projections**
  - Angle $\alpha = 63.7$ degrees or $\arctan(2)$
  - Halves the length of a line segment perpendicular to the projection plane – more realistic than cavalier
https://en.wikipedia.org/wiki/Graphical_projection#/media/File:Graphical_projection_comparison.png
Projections, continued
Perspective projections

- Mimics our human visual system or a camera
- Project in front of the center of projection
- Objects of equal size at different distances from the viewer will be projected at different sizes: nearer objects will appear bigger
Types of perspective projections

• Any set of parallel lines that are not parallel to the projection plane converges to a vanishing point, which corresponds to point at infinity in 3D

• One-, two-, three-point perspective views are based on how many principal axes are cut by projection plane
Vanishing points
Simple perspective projection

- COP at $z = 0$
- Projection plane at $z = d$

\[
\frac{x_p}{d} = \frac{x}{z} \quad x_p = \frac{x}{z/d}
\]

\[
y_p = \frac{y}{z/d}
\]

- Transformation is not invertible or affine.
  Derive the projection matrix.
Simple perspective projection

- How to get a perspective projection matrix?
- Homogeneous coordinates come to the rescue

\[ x_p = \frac{x}{z/d} \]
\[ y_p = \frac{y}{z/d} \]

\[ M_p = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1/d & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z \\
z/d
\end{bmatrix} = \begin{bmatrix}
x \\
\frac{x}{z/d} \\
\frac{y}{z/d} \\
d \\
1
\end{bmatrix} \]
Summary of simple projections

• COP: origin of the coordinate system
• Look into positive z direction
• Projection plane perpendicular to z axis

\[ M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 & \cos \phi / \tan \alpha & 0 \\ 0 & 1 & \sin \phi / \tan \alpha & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \]
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