Tomographic Reconstruction

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Overview

• Tomographic Reconstruction
  – Reconstruction from Projection
  – Computed Tomography (CT)
  – Radon Transform
  – Parallel-Beam Filtered Backprojections
  – Fan-Beam Filtered Backprojections
Tomography

• Derived from the greek words tome (to cut) and graphein (to write)

• Imaging by sectioning (CT, PET)

• Note: Cutting humans into thin slices for diagnostic reasons is generally considered bad practice. This is why we use imaging techniques for that purpose.
Computed Tomography


https://www.radiologyinfo.org/gallery-items/images/abdo-ct-norm.jpg

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CT Scanner Generations

1. **First generation**
   - Diagram: [Image 1](http://tech.snmjournals.org/content/35/3/115/F2.large.jpg)

2. **Second generation**
   - Diagram: [Image 2](http://tech.snmjournals.org/content/35/3/115/F7.large.jpg)

3. **Third generation**
   - Diagram: [Image 3](http://tech.snmjournals.org/content/35/3/115/F8.large.jpg)

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CT Scanner Generations

fourth generation

fifth generation

sixth generation

http://tech.snmjournals.org/content/35/3/115/F10.large.jpg

https://wiki.uiowa.edu/download/attachments/40534397/Fourth%20Generation%20scanner.jpg


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CT Scanner Generations

seventh generation

Reconstruction from Projections

Figure 5.32
(a) Flat region showing a simple object, an input parallel beam, and a detector strip.
(b) Result of back-projecting the sensed strip data (i.e., the 1-D absorption profile).
(c) The beam and detectors rotated by 90°.
(d) Back-projection.
(e) The sum of (b) and (d). The intensity where the back-projections intersect is twice the intensity of the individual back-projections.

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Backprojections

FIGURE 5.33
(a) Same as Fig. 5.32(a).
(b)–(e) Reconstruction using 1, 2, 3, and 4 backprojections 45° apart.
(f) Reconstruction with 32 backprojections 5.625° apart (note the blurring).

Backprojections

**FIGURE 5.34** (a) A region with two objects. (b)–(d) Reconstruction using 1, 2, and 4 backprojections 45° apart. (e) Reconstruction with 32 backprojections 5.625° apart. (f) Reconstruction with 64 backprojections 2.8125° apart.
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Radon Transform

\[ x \cos \theta + y \sin \theta = \rho \]

**FIGURE 5.36** Normal representation of a straight line.


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Radon Transform

A point $g(\rho_j, \theta_k)$ in the projection

Complete projection, $g(\rho, \theta_k)$, for a fixed angle

FIGURE 5.37
Geometry of a parallel-ray beam.
Radon Transform

• Single line integral:

\[ g(\rho_j, \theta_k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta_k + y \sin \theta_k - \rho_j) \, dx \, dy \]

• generalizing gives the projection (line integral) along a line:

\[ g(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - \rho) \, dx \, dy \]

• discrete representation (for discrete values of x and y):

\[ g(\rho, \theta) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \delta(x \cos \theta + y \sin \theta - \rho) \]
Sinograms of Radon Transforms

**FIGURE 5.39** Two images and their sinograms (Radon transforms). Each row of a sinogram is a projection along the corresponding angle on the vertical axis. Image (c) is called the Shepp-Logan phantom. In its original form, the contrast of the phantom is quite low. It is shown enhanced here to facilitate viewing.
Backprojection

• Back-projecting by copying each line $L(\rho_j, \theta_k)$ of the projection $g$ at angle $\theta$ onto the image:

$$f_\theta(x, y) = g(x \cos \theta + y \sin \theta, \theta)$$

• Final image is created by integrating over all back-projected images:

$$f(x, y) = \int_0^\pi f_\theta(x, y) \, d\theta$$

• …which becomes the sum of all images in the discrete case:

$$f(x, y) = \sum_{\theta=0}^{\pi} f_\theta(x, y)$$
Backprojections

FIGURE 5.40
Backprojections of the sinograms in Fig. 5.39.


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Fourier-Slice Theorem

**FIGURE 5.41** Illustration of the Fourier-slice theorem. The 1-D Fourier transform of a projection is a slice of the 2-D Fourier transform of the region from which the projection was obtained. Note the correspondence of the angle $\theta$. 

Filtered Backprojection

FIGURE 5.42
(a) Frequency domain plot of the filter $|\omega|$ after band-limiting it with a box filter. (b) Spatial domain representation. (c) Hamming windowing function. (d) Windowed ramp filter, formed as the product of (a) and (c). (e) Spatial representation of the product (note the decrease in ringing).
### FIGURE 5.43
Filtered back-projections of the rectangle using (a) a ramp filter, and (b) a Hamming-windowed ramp filter. The second row shows zoomed details of the images in the first row. Compare with Fig. 5.40(a).
Filtered Backprojection

**FIGURE 5.44**
Filtered backprojections of the head phantom using (a) a ramp filter, and (b) a Hamming-windowed ramp filter. Compare with Fig. 5.40(b).
Filtered Backprojection

• Complete back-projected image is obtained as follows:

1. Compute the 1D Fourier Transform of each projection

2. Multiply each Fourier Transform by the filter function

3. Obtain the inverse 1D Fourier Transform of each resulting filtered transform

4. Integrate (sum) all the 1D inverse transforms
Filtered Backprojection

- Backprojection can also be obtained in the spatial domain.
- Multiplication of projection with the filter in the Fourier domain becomes convolution with the filter in the spatial domain.
- Convolution is in most cases less computational expensive than transformation into Fourier domain.
- Filter properties of Fourier domain filter introduce negative pixel values into backprojection, which have to be handled — spatial domain filter cannot introduce negative pixel values.
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Fan-Beam Filtered Backprojection

Fan-Beam Filtered Backprojection

**FIGURE 5.48**
Reconstruction of the rectangle image from filtered fan backprojections.
(a) $1^\circ$ increments of $\alpha$ and $\beta$.
(b) $0.5^\circ$ increments.
(c) $0.25^\circ$ increments.
(d) $0.125^\circ$ increments.
Compare (d) with Fig. 5.43(b).

Fan-Beam Filtered Backprojection

**FIGURE 5.49**
Reconstruction of the head phantom image from filtered fan backprojections.
(a) 1° increments of α and β.
(b) 0.5° increments.
(c) 0.25° increments.
(d) 0.125° increments.
Compare (d) with Fig. 5.44(b).
